Toward a Nonlinear Acoustic Analogy: Turbulence as a Source of Sound and Nonlinear Propagation

Steven A. E. Miller
The National Aeronautics and Space Administration

NASA Technical Working Group
April 21st-22nd 2015

Based on Miller, S. A. E., "Toward a Nonlinear Acoustic Analogy: Turbulence as a Source of Sound and Nonlinear Propagation," NASA TM, 2015.

Acknowledgements

- NASA Advanced Air Vehicles Program Commercial Supersonic Technology Project
- Brian Howerton – NASA Langley – measurements from NASA Normal Incidence Tube
- Emily Mazur – NASA 2012 Intern - evaluated Blackstock bridging function
- Many previous curious researchers

Introduction

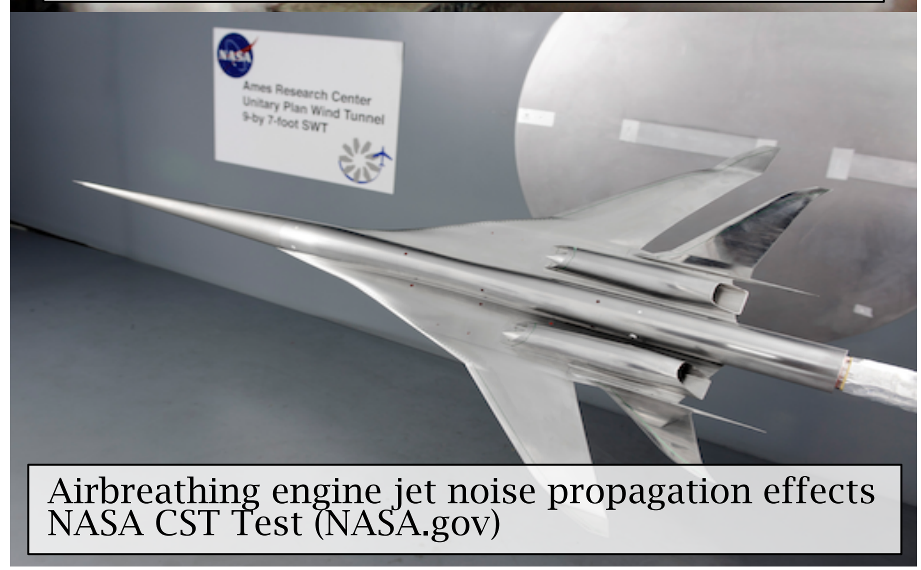
Turbulence and Nonlinear Propagation

- Aerospace vehicles produce turbulence

- Sound propagates nonlinearly if turbulence is highly intense

- Intense noise is harmful to the vehicle and environment

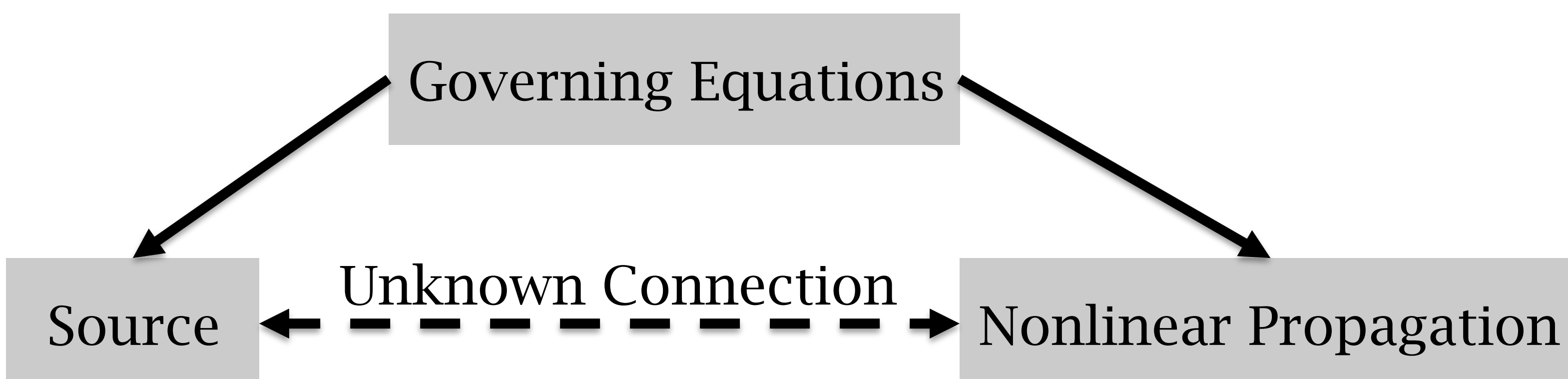
Rocket noise has nonlinear propagation effects
NASA SMAT Test (NASA.gov)



Airbreathing engine jet noise propagation effects
NASA CST Test (NASA.gov)

Turbulence and Nonlinear Propagation

- Understand different mathematical models of sound generation and propagation
- Relate the governing equations to sound generation and propagation
- Show a mathematical connection between sound generation (acoustic analogy) and sound propagation (Burgers' equation)



Mathematical Models

Claude-Louis Navier and George Gabriel Stokes

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0 \qquad \frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = \frac{\partial \tau_{ij}}{\partial x_j}$$

$$\frac{\partial \rho e_o}{\partial t} + \frac{\partial \rho u_j e_o}{\partial x_j} = -\frac{\partial u_j p}{\partial x_j} - \frac{\partial q_j}{\partial x_j} + \frac{\partial u_i \tau_{ij}}{\partial x_j}$$

Navier

Claude-Louis Marie Henri Navier

- 1735-1836
- French
- Professor at École Nationale des Ponts et Chaussées
- Known for elasticity and structural engineering

Sir George Stokes

- 1819-1903
- Irish
- Lucasian Professor
- Fluids, Optics, Chemistry
- Politics and Theology

Stokes

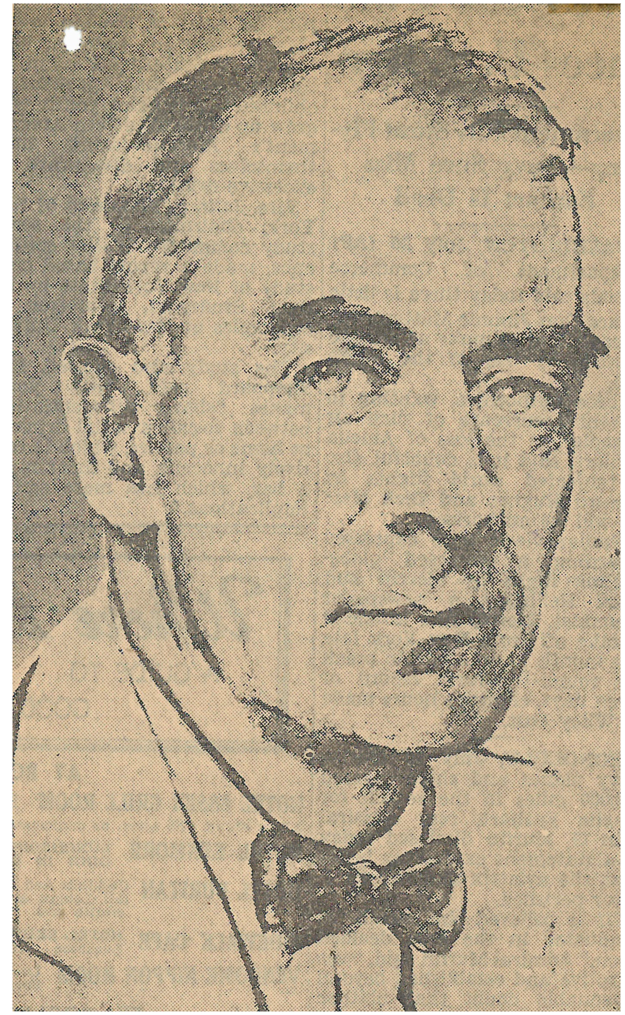
Richard D. Fay

Governing Equation (Conjecture)

$$c_{\infty}^{2}\frac{\partial^{2}y}{\partial x^{2}}=\frac{\partial y}{\partial x}^{\gamma+1}\left[\frac{\partial^{2}y}{\partial t^{2}}-\frac{4\mu}{3\rho_{\infty}}\frac{\partial}{\partial t}\left(\frac{\partial^{2}y}{\partial x^{2}}\right)\right]$$

- Governs shocked one-dimensional finite amplitude waves
- y is particle displacement
- Solution via assumptions
  - Periodic
  - $dy\,/\,dx$ is Fourier series
  - Substitute and solve Fourier series

Fay, R. D., "Plane Sound Waves of Finite Amplitude," Journal of the Acoustical Society of America, Vol. 3, No. 9, 1931, pp. 222–241. doi:10.1121/1.1901928.



Courtesy of the MIT Electrical Engineering and Computer Science Department

Solution

$$\frac{p}{p_{\infty}}=\frac{32}{3}\frac{\mu\omega}{c_{\infty}^{2}\rho_{\infty}}\left(\frac{\gamma}{\gamma+1}\right)\sum_{n=1}^{n=\infty}\frac{\sin n\left(\omega t-\omega x/c_{\infty}\right)}{\sinh n\left[\log\left[\frac{16\mu\omega}{3\rho_{\infty}(\gamma+1)c_{\infty}^{2}K_{1,1}}\right]+\frac{2x\mu\omega^{2}}{3c_{\infty}^{3}\rho_{\infty}}\right]}$$

Guido Fubini-Ghiron

Governing Equation (Conjecture)

$$\frac{\partial^2 \xi}{\partial t^2} + \frac{\partial \xi}{\partial t}\frac{\partial^2 \xi}{\partial x \partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

- Continuous non-conservative one-dimensional finite amplitude waves
- ξ is particle displacement
- Solution via Earnshaw approach
  - Write as binomial series and truncate
  - Convert to Eulerian framework and rewrite as Fourier series

Solution

$$\frac{p}{p_\infty} = \sum_{n=1}^{\infty} \frac{2}{n\sigma} J_n[n\sigma] \sin n\left(\omega t - kx\right)$$

Fubini-Ghiron, G., "Anomalie nella Propagazione di onde Acustiche di Grande Ampiezza," Alta Frequenza, Vol. 4, 1935, pp. 530–581.

Italian, 1879-1943, Professor of Mathematics at Princeton

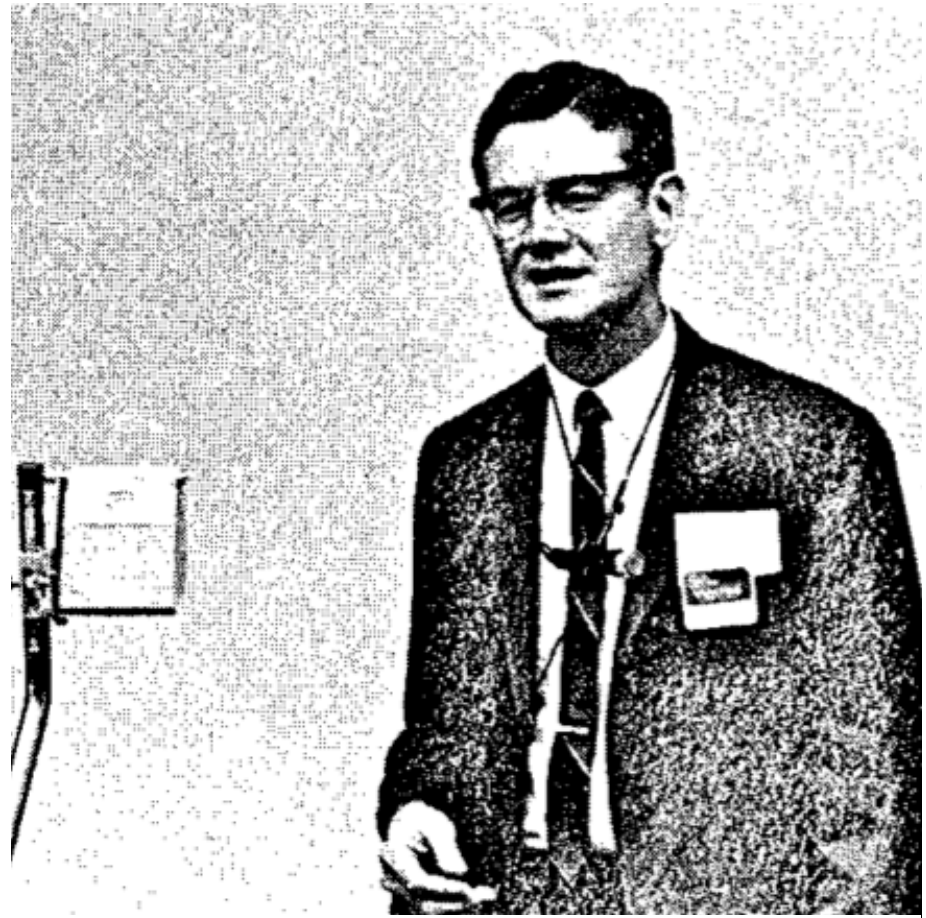
David T. Blackstock

Governing Equations (Conjecture)

$$u = g(\phi) \quad \tau = \phi - (\beta c_\infty^{-2}) g(\phi)$$

$$\frac{dt'_s}{dx} = -\frac{1}{2}\beta c_\infty^{-2}(u_a + u_b)$$

- Weak shock theory
- g is a function and Φ is emission time
- Direct solution approach by substitution after eliminating Φ
    - Assume boundary value problem
    - Resultant transcendental equation solved with Fourier series assumption



Blackstock, D. T., `History of Nonlinear Acoustics and a Survey of Burgers' and Related Equations,' 1969.

Solution

$$p(x,t) = p_o \sum_{n=1}^{\infty} B_n \sin\left[n\omega\tau\right]$$

$$B_n = \frac{2}{n(1+\sigma)} + \frac{2}{n\pi\sigma}\int_{\Phi_{sh}}^{\pi} \cos\left[n\left(\Phi - \sigma\sin\Phi\right)\right] d\Phi$$

Blackstock, D. T., "Connection Between the Fay and Fubini Solutions for Plane Sound Waves of Finite Amplitude," Journal of the Acoustical Society of America, Vol. 39, No. 6, 1965, pp. 1019–1026. doi:10.1121/1.1909986.

M. J. Lighthill

Governing equations are Navier-Stokes

- Exactly rearrange to form a governing equation, the acoustic analogy

$$\frac{\partial^2 \rho}{\partial t^2} - c_\infty^2 \frac{\partial^2 \rho}{\partial x_i \partial x_i} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$$

- Right hand side is equivalent source
- Left hand side is linear wave operator

Lighthill, M. J., "On Sound Generated Aerodynamically. I. General Theory," Proc. R. Soc. Lond. A., Vol. 211, No. 1107, 1952, pp. 564–587. doi:10.1098/rspa.1952.0060.

One solution loosely based on Ffowcs Williams

$$S(\boldsymbol{x},\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{r_i r_j r_l' r_m'}{c_\infty^4 r^2 r'^2} g\left(\boldsymbol{x},\boldsymbol{y},\omega\right) g^*\left(\boldsymbol{x},\boldsymbol{y}',\omega\right) \frac{\partial^4}{\partial \tau^4} R_{ijlm}(\boldsymbol{y},\boldsymbol{\eta},\tau)$$
$$\times \exp\left[-i\omega\left(\tau + \frac{r}{c_\infty} - \frac{r'}{c_\infty}\right)\right] d\tau d\boldsymbol{\eta} d\boldsymbol{y}$$

English (born Paris), 1924-1998, Lucasian Professor at Cambridge

David G. Crighton

Governing equations is Navier-Stokes

- Assume
  - *u is* summation of a gradient and cross-product, eliminate high order terms, flow is irrotational
  - Solutions are set of symmetry
  - *Kr* >> 1, `linear wavenumber'

Governing Equation (non-gen. Burgers')

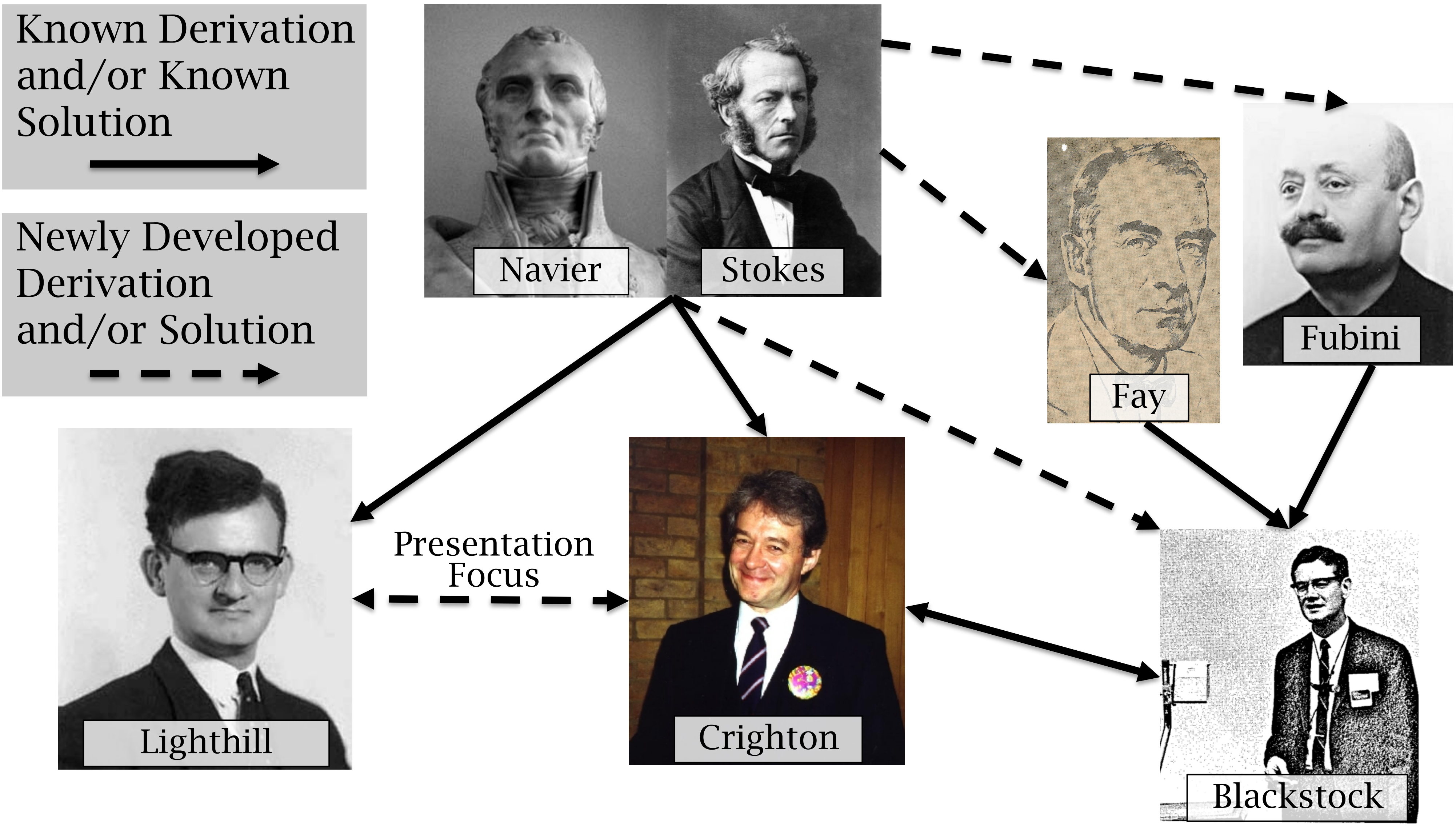
$$\frac{\partial u}{\partial t} + c_\infty \frac{\partial u}{\partial r} + \frac{\gamma+1}{2} u \frac{\partial u}{\partial r} + \frac{j c_\infty u}{2r} = \frac{\delta}{2} \frac{\partial^2 u}{\partial r^2}$$

- Spherical, cylindrical, and planar nonlinear wave propagation

Crighton, D. G., "Model Equations of Nonlinear Acoustics," Annual Review of Fluid Mechanics, Vol. 11, 1979, pp. 11–33. doi:10.1146/annurev.fl.11.010179.000303.

English, 1942-2000, Professor of Applied Mathematics Cambridge
Also Opera lover – so am I! :)

Mathematical Relationships



Miller NASA TM shows the mathematical connections and solutions of ALL relations!

The Navier-Stokes Equations and the Acoustic Analogy

Governing equations are Navier-Stokes.
We now think of the Green's function as satisfying

$$\rho(\boldsymbol{x},t) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} g \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} d\tau d\boldsymbol{y}$$

We can show using the cross-spectral acoustic analogy

$$S(\boldsymbol{x},\omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{r_i r_j r'_l r'_m}{c_\infty^4 r^2 r'^2} g\left(\boldsymbol{x},\boldsymbol{y},\omega\right) g^*\left(\boldsymbol{x},\boldsymbol{y}',\omega\right) \frac{\partial^4}{\partial \tau^4} R_{ijlm}(\boldsymbol{y},\boldsymbol{\eta},\tau)$$
$$\times \exp\left[-i\omega\left(\tau + \frac{r}{c_\infty} - \frac{r'}{c_\infty}\right)\right] d\tau d\boldsymbol{\eta} d\boldsymbol{y}$$

A statistical source model for sound generation (altered from Miller) is

$$\frac{\partial^4}{\partial \tau^4} R_{ijlm}(\boldsymbol{y},\boldsymbol{\eta},\tau) = \frac{4 A_{ijlm} \overline{u}^4}{\pi^{1/2} l_s^8} \left(3 l_s^4 - 12 l_s^2 (\xi - \overline{u}\tau)^2 + 4(\xi - \overline{u}\tau)^4\right)$$
$$\times \exp\left[\frac{-|\xi|}{\overline{u}\tau_s}\right] \exp\left[\frac{-(\xi - \overline{u}\tau)^2}{l_s^2}\right] \exp\left[\frac{-\eta^2}{l_{sy}^2}\right] \exp\left[\frac{-\zeta^2}{l_{sz}^2}\right]$$

Miller, S. A. E., "Prediction of Near-Field Jet Cross Spectra," AIAA Journal, 2015. doi:10.2514/1.J053614.

The Navier-Stokes Equations and the Acoustic Analogy

Using the source model, assuming that the observer is in the far-field, simplifying, and carefully rearranging yields

$$S(\boldsymbol{x},\omega) = \frac{\pi\omega^4}{c_\infty^4} \underbrace{g\left(\boldsymbol{x},\omega\right) g^*\left(\boldsymbol{x},\omega\right)}_{\text{Green's function}} \underbrace{\int\limits_{-\infty}^{\infty} A_{ijlm} \frac{r_i r_j r_l r_m}{r^4} \frac{l_s l_{sy} l_{sz}}{\overline{u}} \exp\left[\frac{-l_s^2\omega^2}{4\overline{u}^2}\right] \times \int\limits_{-\infty}^{\infty} \exp\left[\frac{-i\xi\omega}{\overline{u}}\right] \exp\left[\frac{-|\xi|}{\overline{u}\tau_s}\right] d\xi dy_1}_{\text{Source Spectrum}}$$

Selective far-field assumption

- Source remains a volumetric integral
- Propagation approximated from a point within source volume

Need to find what *gg** is to capture nonlinear propagation effects

The Navier-Stokes Equations and a Burgers' Equation

The Navier-Stokes equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0 \qquad \text{and} \qquad \frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = \frac{\partial \tau_{ij}}{\partial x_j}$$

Following Crighton then finding a more compact form governing pressure

$$\frac{\partial p}{\partial x} + m\frac{p}{r} - \epsilon p \frac{\partial p}{\partial \tau} = \frac{\delta}{2c_\infty^3}\frac{\partial^2 p}{\partial \tau^2}$$

Select analytical solutions exist – eg: Blackstock, Fay, and Fubini

Seek a numerical solution in the frequency domain (as shown by Saxena)

$$\frac{\partial \tilde{p}}{\partial r} + m\frac{\tilde{p}}{r} + (\alpha + i\beta)\,\tilde{p} = \frac{i\omega\epsilon}{2}\tilde{q}$$

Pseudo-spectral numerical method marches solution in space from prescribed boundary condition (same BC as Blackstock)

The Connection Between the Acoustic Analogy and Generalized Burgers' Equation

Conjecture: Given the solution of

$$\frac{\partial \tilde{p}}{\partial r} + m\frac{\tilde{p}}{r} + (\alpha + i\beta)\,\tilde{p} = \frac{i\omega\epsilon}{2}\tilde{q}$$

subject to the boundary condition of a source spectrum of the acoustic analogy and $x >> D$ then

$$g(\boldsymbol{x},\omega)g^*(\boldsymbol{x},\omega) \approx \tilde{p}(\boldsymbol{x},\omega)\tilde{p}^*(\boldsymbol{x},\omega)$$

within the acoustic analogy. As $\lim \tilde{p} \rightarrow \epsilon$

$$g(\boldsymbol{x},\omega)g^*(\boldsymbol{x},\omega) = \tilde{p}(\boldsymbol{x},\omega)\tilde{p}^*(\boldsymbol{x},\omega)$$

for the traditional approach only.

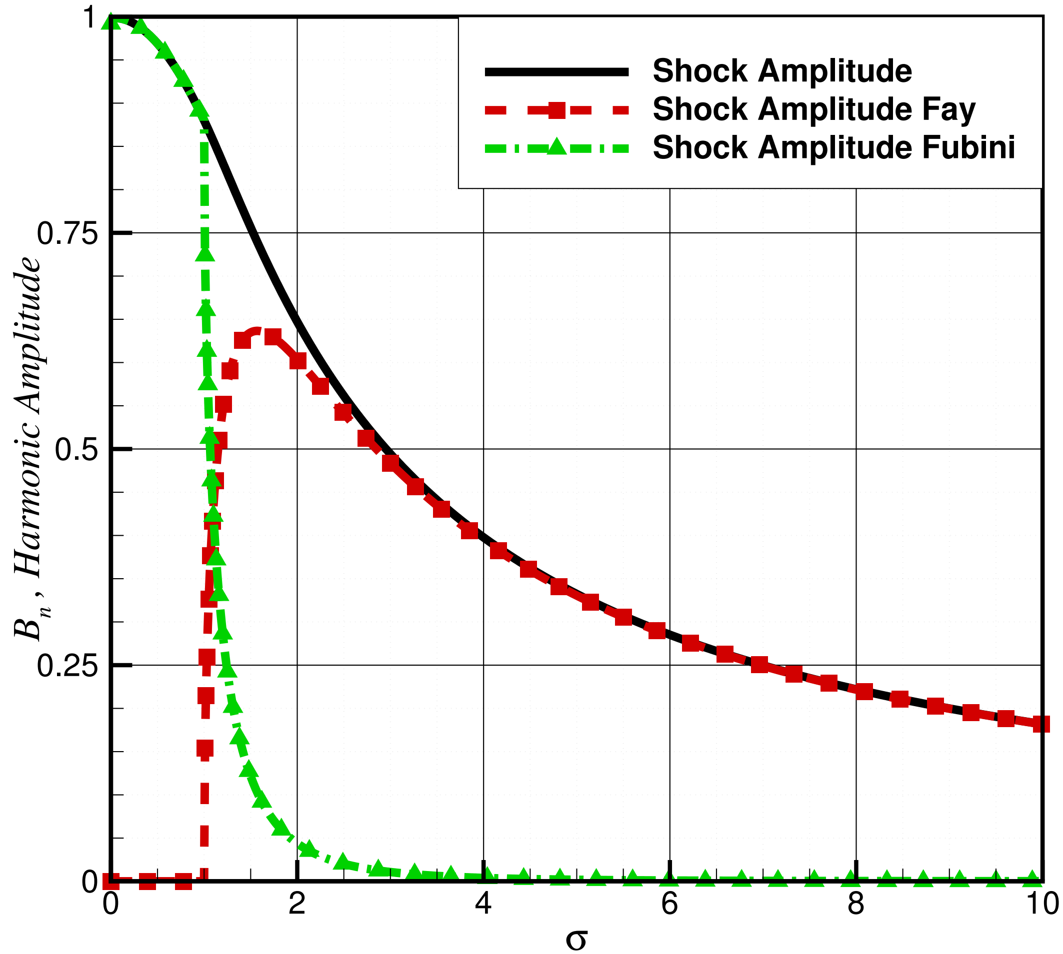
Acoustic Analogy and Burgers' Equation

- Approximation of gg^* is obtained from solution of generalized Burgers' equation
- Boundary condition (at $r = 0$) of generalized Burgers' equation is broadband source spectrum
- Source spectrum at low intensities results in predictions that are equivalent to those produced by traditional acoustic analogies
- Source spectrum at high intensity causes nonlinear terms within generalized Burgers' equation to be dominant
- Characteristics of nonlinear propagation are apparent in predicted jet mixing noise spectrum.

Results

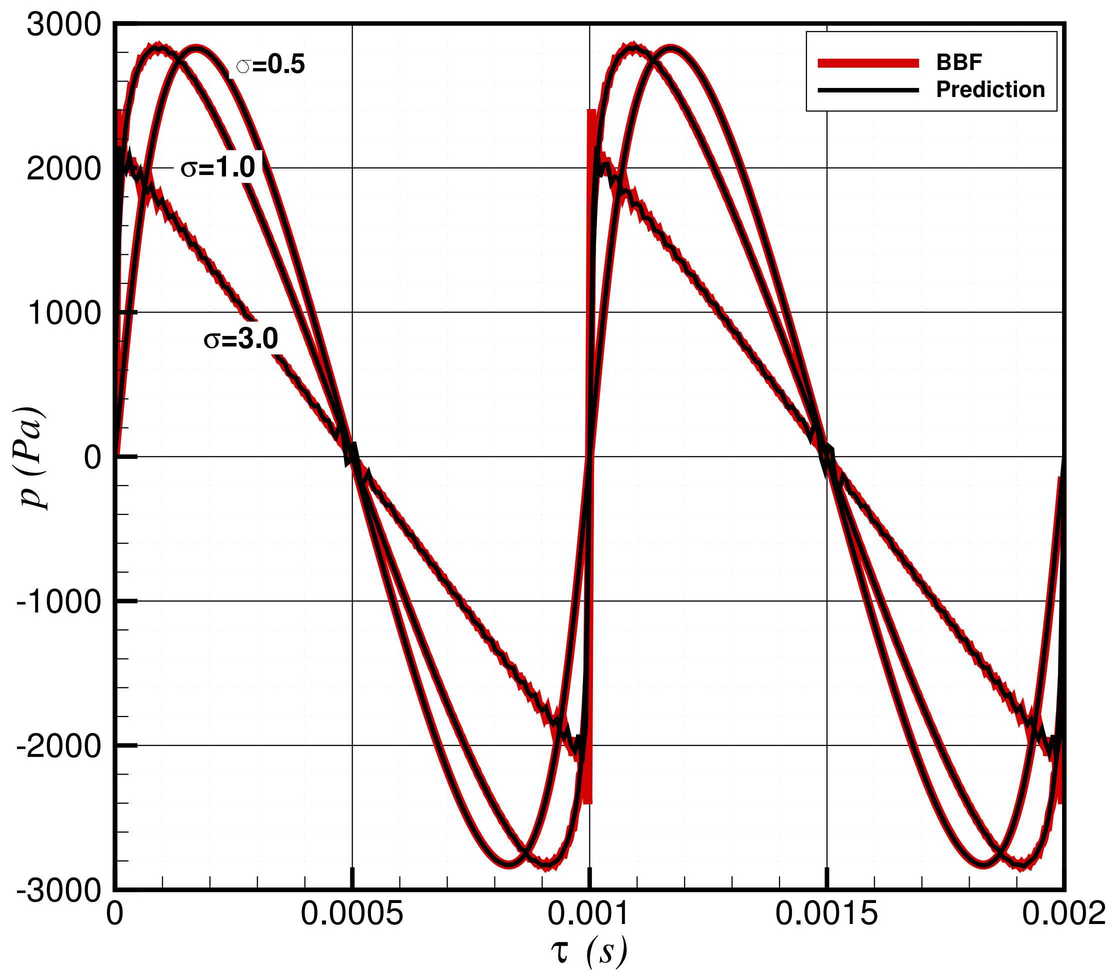
Examination of Blackstock, Fay, and Fubini

- Almost exact solutions of generalized Burgers' equation
- Source planar sin wave at 160dB and 1000Hz
- Regions of validity



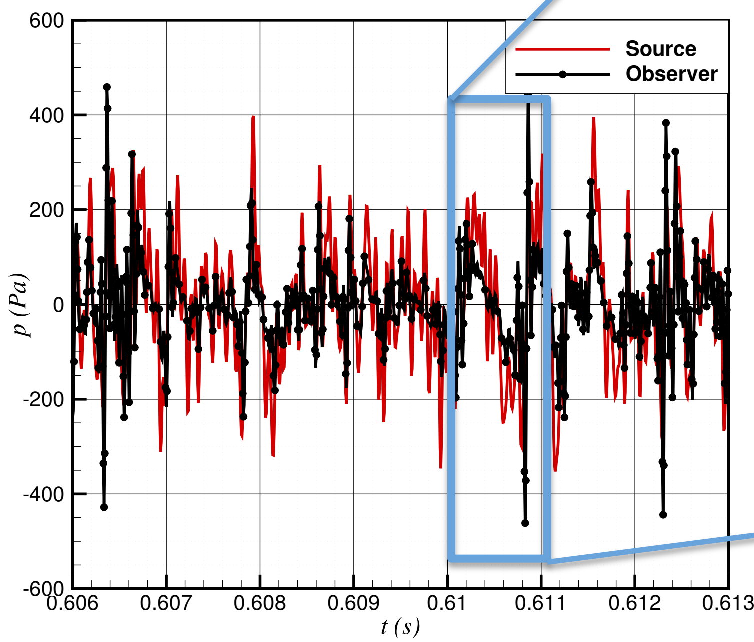
Numerical Solver of Generalized Burgers' Equation and Blackstock Bridging Function

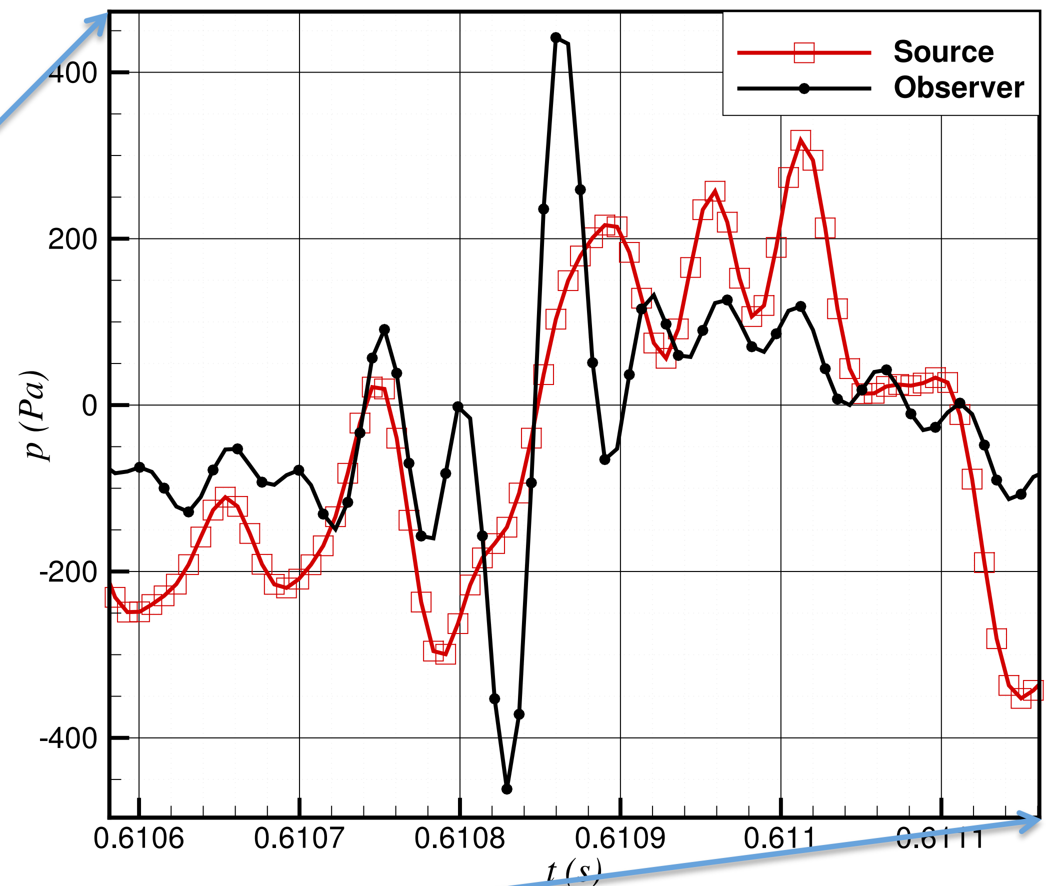
- Comparison at three observer positions
- Numerical solver agrees with analytic result
- Source planar sin wave at 160dB and 1000Hz
- Gibb's phenomenon present



Propagation of a Broadband Signal

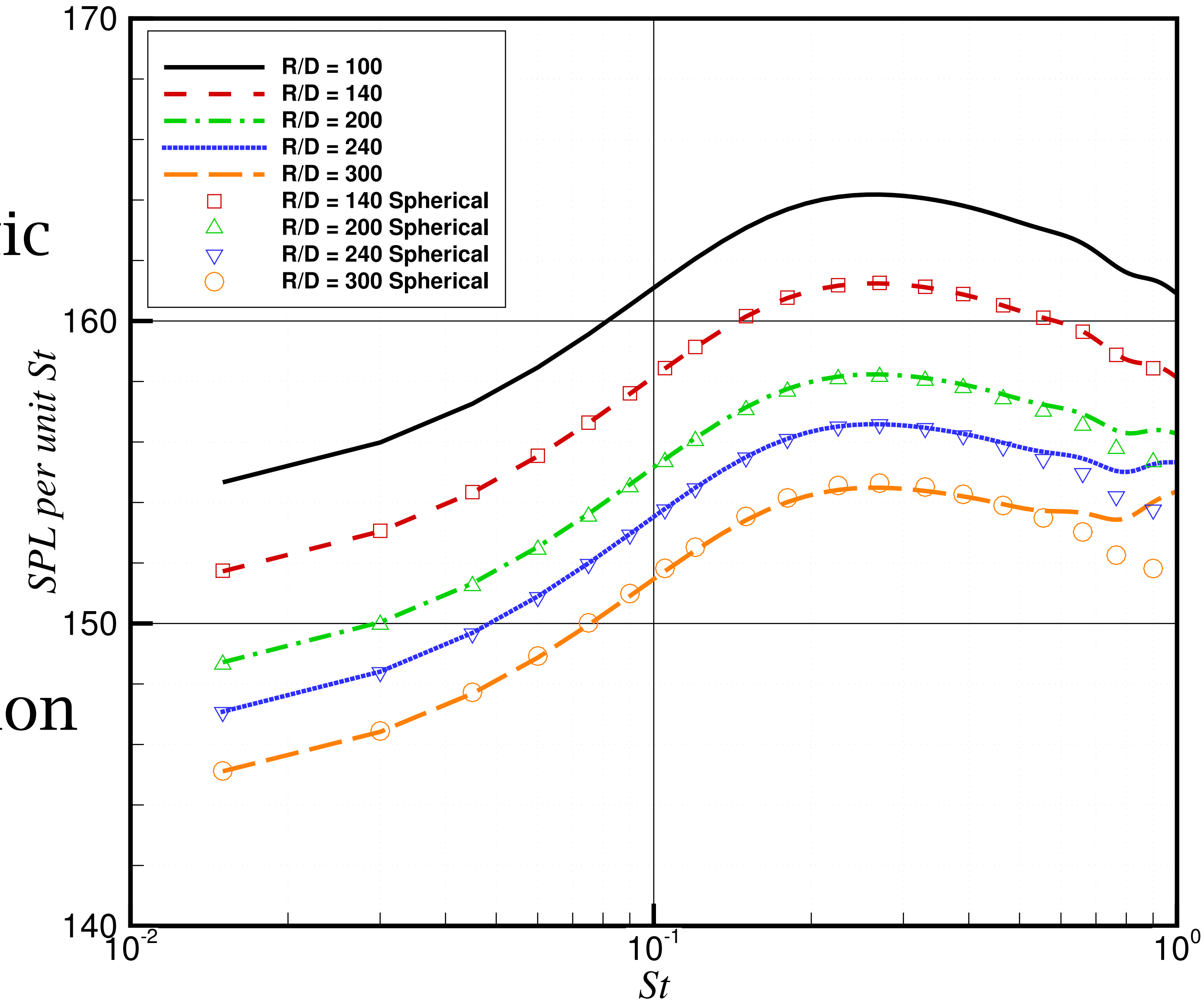
- Shocked observer waveform due to wave coalescence
- Discontinuities not present in source signal
- Not observed in linear acoustics
- Jet $M_j = 1.86$
- Jet TTR = 3.20
- Sideline direction





Example Jet Noise Prediction Directly Incorporating Nonlinear Propagation

- Unified Acoustic Analogy with Nonlinear Propagation
- Jet $M_j = 1.86$
- Jet TTR = 3.20
- Sideline direction



Summary and Conclusion

- Showed connection between Navier-Stokes equations, generalized Burgers' equation (sound propagation), and Acoustic Analogy (sound source)
- Nonlinear propagation taken into account directly from source to observer
- A single equation contains sound source and nonlinear propagation from turbulence
- Evaluated select equations to demonstrate relevant physics

Questions

References

- Neilsen, T. B., Gee, K. L., and James, M. M., "Spectral Characterization in the Near and Mid-Field of Military Jet Aircraft Noise," 19th AIAA/CEAS Aeroacoustics Conference, AIAA Paper 2013-2191, 2013. doi: 10.2514/6.2013-2191.
- Gee, K. L., Shepherd, M. R., Falco, L. E., Atchley, A. A., Ukeiley, L. S., Jansen, B. J., and Seiner, J. M., "Identification of Nonlinear and Near-Field Effects in Jet Noise using Nonlinearity Indicators," 13th AIAA/CEAS Aeroacoustics Conference, AIAA Paper 2007-3653, 2007. doi:10.2514/6.2007-3653.
- Petitjean, B., Viswanathan, K., and McLaughlin, D. K., "Acoustic Pressure Waveforms Mea- sured in High Speed Jet Noise Experiencing Nonlinear Propagation," International Journal of Aeroacoustics, Vol. 5, No. 2, 2006, pp. 193–215. doi:10.1260/147547206777629835.
- Tam, C. K. W. and Parrish, S. A., "Noise of High-Performance Aircrafts at Afterburner," 20th AIAA/CEAS Aeroacoustics Conference, AIAA Paper 2014-2754, 2014. doi:10.2514/6.2014- 2754.
- Morfey, C. L. and Howell, G. P., "Nonlinear Propagation of Aircraft Noise in the Atmosphere," AIAA Journal, Vol. 19, No. 8, 1980, pp. 986–992. doi:10.2514/3.51026.
- McInerny, S. A. and Olcmen, S. M., "High-Intensity Rocket Noise: Nonlinear Propagation, Atmospheric Absorption, and Characterization," Journal of the Acoustical Society of America, Vol. 117, No. 2, 2005, pp. 578–591. doi:10.1121/1.1841711.
- Gee, K. L., Giraud, J. H., Blotter, J. D., and Sommerfeldt, S. D., "Energy-Based Acoustical Measurements of Rocket Noise," 15th AIAA/CEAS Aeroacoustics Conference, AIAA Paper 2009-3165, 2009. doi: 10.2514/6.2009-3165.
- Lighthill, M. J., "On Sound Generated Aerodynamically. I. General Theory," Proc. R. Soc. Lond. A., Vol. 211, No. 1107, 1952, pp. 564–587. doi:10.1098/rspa.1952.0060.
- Miller, S. A. E., "Prediction of Near-Field Jet Cross Spectra," AIAA Journal, 2015. doi:10.2514/1.J053614.
- Schlichting, H. and Gersten, K., "Boundary-Layer Theory," Springer-Verlag, New York, 2000.
- Crighton, D. G., "Model Equations of Nonlinear Acoustics," Annual Review of Fluid Mechanics, Vol. 11, 1979, pp. 11–33. doi:10.1146/annurev.fl.11.010179.000303.

References

- Blackstock, D. T., “Generalized Burgers Equation for Plane Waves,” Journal of the Acoustical Society of America, Vol. 77, No. 6, 1985, pp. 2050–2053. doi:10.1121/1.391778.
- Blackstock, D. T., “History of Nonlinear Acoustics and a Survey of Burgers and Related Equa- tions,” Proceedings of a Conference held at the Applied Research Laboratories, The University of Texas at Austin, 1969, pp. 1–27.
- Vladimirov, V. A. and Maczka, C., “Exact Solutions of Generalized Burgers Equation, De- scribing Travelling Fronts and Their Interaction,” Reports on Mathematical Physics, Vol. 60, No. 2, 2007, pp. 317–328. doi: 10.1016/S0034-4877(07)80142-X.
- Mendousse, J. S., “Nonlinear Dissipative Distortion of Progressive Sound Waves at Moderate Amplitudes,” Journal of the Acoustical Society of America, Vol. 25, No. 51, 1953, pp. 51–54. doi:10.1121/1.1907007.
- Lighthill, M. J., “Viscosity Effects in Sound Waves of Finite Amplitude,” Surveys in Mechanics, Cambridge University Press (Davies R. M. and Batchelor, G. K. (Eds.)), 1956, pp. 250–351.
- Saxena, S., Morris, P. J., and Viswanathan, K., “Algorithm for the Nonlinear Propagation of Broadband Jet Noise,” AIAA Journal, Vol. 47, No. 186-194, 2009, pp. 186–194. doi:10.2514/1.38122.
- Lee, S. L., Morris, P. J., and Brentner, K. S., “Improved Algorithm for Nonlinear Sound Propagation with Aircraft and Helicopter Noise Applications,” AIAA Journal, Vol. 48, No. 11, 2010, pp. 2586–2595. doi: 10.2514/1.J050396.
- Saxena, S., “A New Algorithm for Nonlinear Propagation of Broadband Jet Noise,” M.S. Thesis, The Pennsylvania State University, 2008.
- Duchon, C. E., “Lanczos Filtering in One and Two Dimensions,” Journal of Applied Meteorology, Vol. 18, No. 8, 1979, pp. 1016–1022.
- Bass, H. E., Sutherland, L. C., Zuckerwar, A. J., Blackstock, T. D., and Hester, D. M., “Atmospheric Absorption of Sound: Further Developments,” Journal of the Acoustical Society of America, Vol. 97, No. 1, 1995, pp. 680–683. doi:10.1121/1.412989.

References

- Bass, H. E., Sutherland, L. C., Zuckerwar, A. J., Blackstock, T. D., and Hester, D. M., “Erratum: Atmospheric Absorption of Sound: Further Developments,” Journal of the Acoustical Society of America, Vol. 99, No. 2, 1995, pp. 1259–1259. doi:10.1121/1.415223.
- Fay, R. D., “Plane Sound Waves of Finite Amplitude,” Journal of the Acoustical Society of America, Vol. 3, No. 9, 1931, pp. 222–241. doi:10.1121/1.1901928.
- Blackstock, D. T., “Connection Between the Fay and Fubini Solutions for Plane Sound Waves of Finite Amplitude,” Journal of the Acoustical Society of America, Vol. 39, No. 6, 1965, pp. 1019–1026. doi: 10.1121/1.1909986.
- Fubini-Ghiron, E., “Anomalie nella Propagazione di onde Acustiche di Grande Ampiezza,” Alta Frequenza, Vol. 4, 1935, pp. 530–581.
- Westervelt, P. J., “The Mean Pressure and Velocity in a Plane Acoustic Wave in a Gas,” The Journal of the Acoustical Society of America, Vol. 22, No. 3, 1950, pp. 319–327. doi:10.1121/1.1906606.
- Earnshaw, S., “On the Mathematical Theory of Sound,” Phil. Trans. R. Soc. London, Vol. 150, 1860, pp. 133–148. doi:10.1098/rstl.1860.0009.
- Ffowcs Williams, J. E., “The Noise from Turbulence Convected at High Speed,” Phil. Trans. R. Soc. Lond. A, Vol. 255, No. 1063, 1963, pp. 469–503. doi:10.1098/rsta.1963.0010.
- Lau, J. C., Morris, P. J., and Fisher, M. J., “Measurements in Subsonic and Supersonic Free Jets using a Laser Velocimeter,” Journal of Fluid Mechanics, Vol. 93, No. 1, 1979, pp. 1–27. doi:10.1017/S0022112079001750.
- Lau, J. C., Morris, P. J., and Fisher, M. J., “Effects of Exit Mach Number and Temperature on Mean-Flow and Turbulence Characteristics in Round Jets,” Journal of Fluid Mechanics, Vol. 105, No. 1, 1981, pp. 193–218. doi: 10.1017/S0022112081003170.
- Tam, C. K. W., “Broadband Shock-Associated Noise of Moderately Imperfectly Expanded Jets,” Journal of Sound and Vibration, Vol. 140, No. 1, 1990, pp. 55–71. doi:10.1016/0022- 460X(90)90906-G.
- Schultz, T., Liu, F., Cattafesta, L., Sheplak, M., and Jones, M., “A Comparison Study of Normal-Incidence Acoustic Impedance Measurements of a Perforate Liner,” 15th AIAA/CEAS Aeroacoustics Conference, AIAA Paper 2009-3301, 2009. doi:10.2514/6.2009-3301.